Directional Lipschitzness of Minimal Time Functions in Hausdorff Topological Vector Spaces

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Resumo

In a general Hausdorff topological vector space E, we associate to a given nonempty closed set $S \subset E$ and a bounded closed set $\Omega \subset E$, the minimal time function $T_{S,\Omega}$ defined by $T_{S,\Omega}(x) := inf\{t > 0 : S \cap (x + t\Omega) \neq \emptyset\}$. The study of this function has been the subject of various recent works (see [3, 4, 5, 6, 8, 9, 10] and the references therein). The main objective of this work is in this vein. We characterize, for a given Ω, the class of all closed sets S in E for which $T_{S,\Omega}$ is directionally Lipschitz in the sense of Rockafellar [12]. Those sets S are called Ω-epi-Lipschitz. This class of sets covers three important classes of sets: epi-Lipschitz sets introduced in [12], compactly epi-Lipschitz sets introduced in [2], and K-directional Lipschitz sets introduced recently in [7]. Various characterizations of this class have been established. In particular, we characterize the Ω-epi-Lipschitz sets by the nonemptiness of a new tangent cone, called Ω-hypertangent cone. As for epi-Lipschitz sets in Rockafellar [11] we characterize the new class of Ω-epi-Lipschitz sets with the help of other cones. The spacial case of closed convex sets is also studied. Our main results extend various existing results proved in [1, 7] from Banach spaces and normed spaces to Hausdorff topological vector spaces.

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